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Resolution of the Strong CP Problem

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It is shown that the quark mass aligns QCD θ vacuum in such a way that the strong CP is conserved, resolving the strong CP problem.

Quantum chromodynamics (QCD) is well established as the fundamental theory for the strong interactions. However, there has been a persistent puzzle with QCD, namely the smallness of the strong CP violation. The most general QCD Lagrangian is given in the form:

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{\theta_0} + \mathcal{L}_m, \tag{1}$$

where

$$\begin{aligned}
\mathcal{L}_0 &= -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} \\
&\quad +i\sum_i \left[\bar{\psi}_{L,i}(\partial\!\!\!/ + ig\, A\!\!\!/^a T^a)\psi_{L,i} + (L \to R)\right], \\
\mathcal{L}_{\theta_0} &= \frac{\theta_0}{32\pi^2}F^a_{\mu\nu}\tilde{F}^{a\mu\nu}, \\
\mathcal{L}_m &= \sum_{ij} m_{ij}\bar{\psi}_{L,i}\psi_{R,j} + \text{H.c.}
\end{aligned} \tag{2}$$

As usual $F^a_{\mu\nu}$ denotes the field strength tensor for the gluons A^a_μ, g and θ_0 are coupling constants, and $\psi_{L(R),i} = \frac{1}{2}(1 \mp \gamma_5)\psi_i, i = 1, \cdots, \mathrm{N}_f$, denote the N_f quark flavors. The quark mass m_{ij} can be an arbitrary complex matrix, but using the $\mathrm{SU(N}_f)_L \times \mathrm{SU(N}_f)_R$ chiral symmetry of $\mathcal{L}_0$ can be written without loss of generality as

$$m_{ij} = m_i \delta_{ij} e^{i\delta}, \tag{3}$$

where m_i are real and positive, and δ is a constant phase. As well known, the axial U(1) anomaly allows one to shift the phase δ into $\mathcal{L}_{\theta_0}$ and vice versa [1]. This property can be used to remove $\mathcal{L}_{\theta_0}$, and write the QCD Lagrangian (1) as

$$\tilde{\mathcal{L}} = \mathcal{L}_0 + \mathcal{L}'_m, \tag{4}$$

where

$$\mathcal{L}'_m = \sum_i m_i \left(e^{i\bar{\delta}}\bar{\psi}_{L,i}\psi_{R,i} + \text{H.c.}\right) \tag{5}$$

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with $\bar{\delta} = \delta + \theta_0/\mathrm{N}_f$. The two theories (1) and (4) are completely equivalent.

Apparently, with nonzero $\bar{\delta}$ the theory (4) appears to break CP. The experimental bound on neutron electric dipole moment suggests $\bar{\delta}$ be extremely small: $\bar{\delta} < 10^{-9}$ [2,3]. Why is $\bar{\delta}$ so small? This is the strong CP problem (for review see [4]).

A small number, unless protected by symmetry, demands for its smallness a *natural* explanation that does not require fine tuning. Among the various solutions proposed for the problem, the most popular is the Peccei-Quinn mechanism [5]. It predicts a very light and weakly interacting particle, the axion [6,7]. Axion, however, has not been observed, and the current window for its mass is quite narrow, ranging from 10^{-6}eV to 10^{-3}eV [8].

In this Letter we show that there is a mechanism within QCD that renders the strong CP conservation *automatic*. The mechanism we propose relies on the QCD θ vacua and the isospin singlet, pseudoscalar meson η'.

Let us assume for the moment that $\mathcal{L}'_m = 0$, i.e., quarks are massless (massless QCD). Then the axial U(1) is broken only by the Adler-Bell-Jackiw anomaly [9]

$$\partial_\mu J_5^\mu = \frac{g^2 \mathrm{N}_f}{16\pi^2} F^a_{\mu\nu} \tilde{F}^{a\mu\nu} = -\partial_\mu K^\mu, \tag{6}$$

where

$$J_5^\mu = \sum_i \bar{\psi}_i \gamma^\mu \gamma_5 \psi_i \tag{7}$$

is the axial current, and

$$K^\mu = -\frac{g^2 \mathrm{N}_f}{16\pi^2} \epsilon^{\mu\alpha\beta\gamma} A^a_\alpha (F^a_{\beta\gamma} - \frac{1}{3}\epsilon^{abc} A^b_\beta A^c_\gamma) \tag{8}$$

is the gauge-dependent topological current. This anomaly equation gives rise to a gauge-dependent conserved charge $\tilde{Q}_5$:

$$\tilde{Q}_5 = \int (J_5^0 + K^0)(\vec{x}, t) d^3\vec{x}. \tag{9}$$

The symmetry of the massless QCD under the rotation

$$U_5(\theta) = e^{i\theta \tilde{Q}_5} \tag{10}$$

can be used to construct a continuum of equivalent vacua. Let $|\Omega_0\rangle$ be a vacuum of the massless QCD. Since at low energies the chiral symmetry $\mathrm{SU(N}_f)_L \times \mathrm{SU(N}_f)_R$ is spontaneously broken to $\mathrm{SU(N}_f)_{L+R}$, $|\Omega_0\rangle$ must belong to the chiral vacua $[\mathrm{SU(N}_f)_L \times \mathrm{SU(N}_f)_R]/\mathrm{SU(N}_f)_{L+R}$. Now, the state $|\theta\rangle$ defined by

$$|\theta\rangle = U_5(\theta)|\Omega_0\rangle \tag{11}$$

can also be a vacuum, equivalent to $|\Omega_0\rangle$, since $\tilde{Q}_5$ commutes with the Hamiltonian of the massless QCD. Note that $|\theta\rangle$ does not belong to the physical Hilbert space built on the vacuum $|\Omega_0\rangle$ because $\tilde{Q}_5$ is not gauge invariant. Therefore, there is a

continuum of vacua, the θ vacua [10,11]. Hence, the continuum of the vacua for the massless QCD is given as the product of the θ vacua and the chiral vacua. Any point in the continuum can be taken as the vacuum, and the physics is oblivious of the particular choice.

Let us now consider the transformation of η' under the $U_5(\theta)$. Since η' couples to the axial current J_5^μ, $\exp(i\eta'/f_{\eta'})$ transforms as $\sum_i \bar{\psi}_{L,i}\psi_{R,i}$ under $U_5(\theta)$. This gives

$$U_5(\theta)^\dagger \eta' U_5(\theta) = \eta' + 2f_{\eta'}\theta \tag{12}$$

because

$$U_5(\theta)^\dagger \sum_i \bar{\psi}_{L,i}\psi_{R,i} U_5(\theta) = e^{2i\theta}\sum_i \bar{\psi}_{L,i}\psi_{R,i}. \tag{13}$$

The η' decay constant $f_{\eta'}$ is a constant of dimension one. Using Eq. (12) we obtain

$$\begin{aligned}\langle\theta+\delta\theta|\eta'|\theta+\delta\theta\rangle &= \langle\theta|U_5(\delta\theta)^\dagger\eta' U_5(\delta\theta)|\theta\rangle \\ &= \langle\theta|\eta'|\theta\rangle + 2f_{\eta'}\delta\theta.\end{aligned} \tag{14}$$

And using Eq. (13) we can also write the quark condensates in the vacuum $|\theta\rangle$ †as

$$\langle\theta|\bar{\psi}_{L,i}\psi_{R,j}|\theta\rangle = |\Delta|\Sigma_{ij}^{(0)}e^{i\Phi_0}, \tag{15}$$

where $\Phi_0 = 2\theta + \phi_0$, with ϕ_0 being a constant phase, and Δ is a constant of dimension three while $\Sigma^{(0)} \in \mathrm{SU(N}_f)$. Thus the phase of the quark condensates tells which θ vacuum the system is in.

We now have all the tools to present our mechanism. For our purpose we can ignore heavy quarks and keep only the first three flavors ($\mathrm{N}_f = 3$). Let us now turn on a *small* quark mass, so that $\mathcal{L}'_m$ can be regarded as a perturbation to $\mathcal{L}_0$. Let $|\Omega_m\rangle$ be the vacuum of the theory (4). Since the isospin breaking by the quark mass is small we can write the quark condensates in leading order as

$$\langle\Omega_m|\bar{\psi}_{L,i}\psi_{R,j}|\Omega_m\rangle = |\Delta|\Sigma_{ij}^{(m)}e^{i\Phi_m}, \tag{16}$$

where $\Sigma^{(m)} \in \mathrm{SU}(3)$ and Φ_m is a constant phase.

Our crucial observation is that the phase Φ_m as well as $\Sigma^{(m)}$ must be determined dynamically, and consequently that they will depend on the quark mass (m_i as well as the phase $\bar{\delta}$). It is not surprising to expect this, since the vacuum $|\Omega_m\rangle$ would in general depend on the parameters of the theory. It is, in fact, well known that $\Sigma^{(m)}$ is determined dynamically through the Dashen's theorem [12], and is dependent on the quark mass. This mechanism is called chiral vacuum alignment. We shall show that Φ_m is also determined dynamically.

Note that because the phase of the quark condensates before the perturbation $\mathcal{L}'_m$ is turned on is dependent on the θ vacuum chosen (Eq. (15)), the dynamical

†This vacuum should really be regarded as a point in the continuum of the vacua for the massless QCD; for notational convenience, only the θ component is made explicit whereas the chiral component is suppressed.

determination of the phase Φ_m implies dynamical selection of the θ vacuum by the quark mass—θ vacuum alignment—exactly in the manner the quark mass selects the chiral vacuum.

Before we show that the quark mass actually aligns the θ vacuum, we observe that with no θ vacuum alignment the theory (4) is inherently ambiguous, in that the physics cannot be determined completely in terms of the parameters of the theory. To see this, we shall assume that there is no θ vacuum alignment, and that $|\theta\rangle$ with the quark condensates (15) was the vacuum of the massless QCD before the quark mass was turned on. To correctly describe the quark mass effects one must first find the true chiral vacuum of the theory [12,2]. Dashen's theorem requires the potential, which lifts the degeneracy of the chiral vacua,

$$\begin{aligned} V(\Sigma^{(0)}) &= \langle\theta|\delta\mathcal{H}|\theta\rangle \\ &= -\langle\theta|\mathcal{L}_m'|\theta\rangle \\ &= -2|\Delta|\mathrm{Re}\left[e^{i(\bar{\delta}+\Phi_0)}\sum_i m_i\Sigma_{ii}^{(0)}\right], \end{aligned} \tag{17}$$

where $\delta\mathcal{H}$ is the perturbation to the Hamiltonian density, be minimized over the variable $\Sigma^{(0)}$. In general the true chiral vacuum that minimizes $V(\Sigma^{(0)})$, will depend on the phase $\bar{\delta}+\Phi_0$, and consequently the physics, in particular CP violation, that depends on $\bar{\delta}$ will arise only through the combined phase $\bar{\delta}+\Phi_0$. Since Φ_0 is not a parameter that appears in the Lagrangian (4), and depends entirely on our choice of the θ vacuum, the physics is ambiguous.‡

This ambiguity can also be demonstrated in the two dimensional Schwinger model [13]:

$$\begin{aligned} \mathcal{L} &= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\left[\bar{\psi}_L(\partial\!\!\!/ + ie A\!\!\!/)\psi_L + (L\to R)\right] \\ &\quad + m(e^{i\delta}\bar{\psi}_L\psi_R + \mathrm{H.c.}), \end{aligned} \tag{18}$$

where A_μ is the U(1) gauge field, m is the fermion mass, and δ is a constant phase. In this model the axial U(1) is anomalous, and therefore a continuum of θ vacua can be constructed in a similar manner as in the massless QCD [10]. Like $\mathcal{L}_{\theta_0}$ in QCD, a topological term $\epsilon_{\mu\nu}F^{\mu\nu}$, where $\epsilon_{\mu\nu}$ is a constant antisymmetric tensor, may be added to the Lagrangian (18), but as before it can be removed by a proper axial U(1) rotation of the fermion field. The phase δ then plays the role of $\bar{\delta}$ in $\mathcal{L}_m'$ of QCD.

‡It is worthwhile to note that the generally accepted, CP violating effective Lagrangian by Baluni [2] is in fact ambiguous. In his derivation of the effective Lagrangian he erroneously put $\Phi_0 = 0, \pi$ and $\Sigma_{ij}^{(0)} = \delta_{ij}$ by requiring that the quark condensates (15) be real so that the vacuum is CP even. However, we note that there is no reason to require the vacuum be CP symmetric, and furthermore the apparent CP violation by complex quark condensates in massless QCD is a fictitious one because the physics is independent on the choice of a particular θ vacuum. Therefore, the Baluni's effective Lagrangian must depend on the arbitrary phase Φ_0. Note that even if one accepts his argument there is still an ambiguity of choosing Φ_0 between the two values, 0 and π.

The easiest way to see the ambiguity is by bosonization of the Lagrangian (18) using the standard rule [14]

$$
\begin{aligned}
\bar\psi i\,\partial\!\!\!/\psi &= \frac{1}{2}(\partial_\mu\sigma)^2\\
\bar\psi\gamma^\mu\psi &= \frac{1}{\sqrt{\pi}}\epsilon^{\mu\nu}\partial_\nu\sigma\\
\bar\psi_L\psi_R &= \frac{e}{2}\exp(i\sqrt{4\pi}\sigma).
\end{aligned} \tag{19}
$$

Note that this bosonization has an inherent ambiguity: the freedom to shift $\sigma\rightarrow\sigma+\sigma_0$, where σ_0 is an arbitrary constant. Because this ambiguity affects only the last equation in (19), and consequently the fermion condensate, it is easy to see that the ambiguity corresponds to a particular selection of the θ vacuum. With this freedom taken into account, the bosonized Lagrangian is given as:

$$
\begin{aligned}
\mathcal{L} \;=\; & -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+\frac{1}{2}(\partial_\mu\sigma)^2-\frac{e}{\sqrt{\pi}}\sigma\epsilon^{\mu\nu}\partial_\mu A_\nu\\
& +m\cos[\delta+\sqrt{4\pi}(\sigma+\sigma_0)],
\end{aligned} \tag{20}
$$

which clearly shows that the δ dependence occurs through $\delta+\sqrt{4\pi}\sigma_0$, and so ambiguous because it depends on an arbitrary parameter. Now, what is the implication of this ambiguity? It implies that σ_0, and accordingly the θ vacuum, must be determined dynamically because the theory (18) cannot be ambiguous. It will be shown shortly that the θ vacuum alignment determines σ_0 dynamically and removes this ambiguity.

We now show that the quark mass in fact induces θ vacuum alignment. As before let us assume that $|\theta\rangle$ was the vacuum of the massless QCD before the quark mass was turned on, and that the quark condensates were given by (15).

The η' in the vacuum $|\theta\rangle$ can be described by an effective Lagrangian:

$$
\mathcal{L}_{\eta'}=\frac{1}{2}(\partial_\mu\eta')^2-\frac{1}{2}m_{\eta'}^2\eta'^2+\text{interactions}, \tag{21}
$$

where the ignored terms involve cubic or higher powers of the fields, and $m_{\eta'}$ is the η' mass in massless QCD induced by the nonvanishing topological susceptibility [15]. With this Lagrangian, the η' satisfies

$$
\langle\theta|\eta'|\theta\rangle=0. \tag{22}
$$

Upon turning on the quark mass, $\mathcal{L'}_m$ induces the following interaction to $\mathcal{L}_{\eta'}$:

$$
\begin{aligned}
&\sum_i m_i\,|\Delta|\Sigma_{ii}^{(0)}e^{i(\bar\delta+\Phi_0)}e^{i\eta'/f_{\eta'}}+\text{H.c.}\\
&=\text{const.}-2|M\Delta|\left[\sin(\bar\delta+\Phi_0+\phi_{\Sigma^{(0)}})(\eta'/f_{\eta'})\right.\\
&\left.+\frac{1}{2}\cos(\bar\delta+\Phi_0+\phi_{\Sigma^{(0)}})(\eta'/f_{\eta'})^2\right]+O(\eta'^3),
\end{aligned} \tag{23}
$$

where $|M|$ and $\phi_{\Sigma^{(0)}}$ are defined through

$$\sum_i m_i \Sigma^{(0)}_{ii} = |M| e^{i\phi_{\Sigma^{(0)}}}. \tag{24}$$

The effective Lagrangian (21) is thus modified by the quark mass as

$$\begin{aligned}\tilde{\mathcal{L}}_{\eta'} \quad = \quad & \frac{1}{2}(\partial_\mu \eta')^2 - \frac{1}{2}\tilde{m}^2_{\eta'}\eta'^2 \\ & -2|M\Delta| \sin(\bar{\delta} + \Phi_0 + \phi_{\Sigma^{(0)}})(\eta'/f_{\eta'}) \\ & + \text{interactions},\end{aligned} \tag{25}$$

where

$$\tilde{m}^2_{\eta'} = m^2_{\eta'} + 2|M\Delta| \cos(\bar{\delta} + \Phi_0 + \phi_{\Sigma^{(0)}})/f^2_{\eta'}. \tag{26}$$

With the presence of a term linear in η' (for θ satisfying $\sin(\bar{\delta}+\Phi_0+\phi_{\Sigma^{(0)}}) \neq 0$) this equation shows that the quark mass exerts a force on η' and pushes it away from its stable position at $\eta' = 0$; and, therefore, the equation (22) no longer holds. With the help of Eq. (14) we can then see that this shift in the vacuum expectation value of η' implies a realignment of the QCD system to a new θ vacuum. For instance, when the vacuum expectation value of η' shifts from zero to a nonzero value, say δv, the system rotates by $\delta\theta = \delta v/2f_{\eta'}$. The θ vacuum thus becomes unstable in presence of the quark mass, and realigns successively to a new θ vacuum, through the interaction of η' to the quark mass, until the linear term in (25) vanishes. Note that this phenomenon is not much different from the realignment of a ferromagnetic system at the introduction of an external magnetic field.

An important aspect of the Lagrangian (25) is that it should be regarded as valid only for an infinitesimal η' except when the vacuum $|\theta\rangle$ is the stable one. Given a Lagrangian, one would usually minimize the potential of the Lagrangian to find the vacuum and do perturbation around it to read off particle spectrum and interactions. However, with the Lagrangian (25) this would give wrong physics. For instance, a straightforward minimization of the potential in (25) would suggest the θ vacuum rotate only by an amount $\delta\theta \propto |M\Delta|/(\tilde{m}^2_{\eta'} f^2_{\eta'}) \sin(\bar{\delta}+\Phi_0+\phi_{\Sigma^{(0)}})$ whereas according to our argument above the system actually should rotate until $\sin(\bar{\delta}+\Phi_0+\phi_{\Sigma^{(0)}}) = 0$ is satisfied; And also it would give the η'-mass by the unacceptable formula (26) which is ambiguous. The reason that the usual procedure fails with the Lagrangian (25) is that the quark mass induced term (23) is linked to the condition (22); As soon as the system aligns to a new θ vacuum, the Eq. (22) no longer holds, and accordingly the quark mass induced term should be modified through the shift in the phase Φ_0. This makes the effective Lagrangian for η' dependent on the θ vacuum. As an example, when the system rotates from $|\theta\rangle$ to $|\theta + \delta\theta\rangle$, the Lagrangian for η' in the new vacuum $|\theta + \delta\theta\rangle$ is given by (25), but now with $\Phi_0 = 2(\theta + \delta\theta) + \phi_0$. In the usual, spontaneously broken case, this shift in the phase can be absorbed by making a shift in the the associated Nambu-Goldstone boson field, leaving the Lagrangian invariant; consequently, the vacuum alignment in this case is equivalent

to the familiar picture of the rolling of the Nambu-Goldstone boson field from an unstable vacuum to the stable one. However, in the case of η', the rotation of the QCD system cannot be interpreted as the rolling of η' because the nonzero η' mass in the massless QCD does not allow one to absorb $\delta\Phi_0$ by making a shift in the η' field. Therefore, physical quantities like the η' mass can be read off correctly only from the effective Lagrangian written after the system settled down on the stable θ vacuum.

One may recall at this moment the general belief that the θ vacuum is stable against perturbation. The essential point of the argument for the θ vacuum stability [16] is that there is no Nambu-Goldstone boson associated with the symmetry $U_5(\theta)$. In the usual, spontaneously broken case the symmetry breaking term lifts the degeneracy of the vacua and also couples to the associated Nambu-Goldstone bosons. An unstable vacuum thus decays to the stable one by emitting the Nambu-Goldstone bosons. Of course, an identical process cannot happen in the θ vacua because there is no associated Nambu-Goldstone boson. However, we must realize that the absence of the Nambu-Goldstone boson does not prevent the θ vacuum from decaying. As we can see in (23), the quark mass term, which breaks the $U_5(\theta)$ symmetry, couples to η', and thus an unstable θ vacuum can decay through the emission of η's. In this case, however, the decay rate of the unstable vacuum would be slower than the usual case because of the large η' mass.

Now, because the θ vacuum becomes dynamical in presence of the quark mass the true QCD vacuum $|\Omega_m\rangle$ in (16) can be picked up from the continuum of the vacua of the massless QCD (θ vacua times chiral vacua) by minimizing the potential

$$\begin{aligned} V(\Phi,\Sigma) &= -\langle\Omega|\mathcal{L}'_m|\Omega\rangle \\ &= -2|\Delta|\mathrm{Re}\left[e^{i(\bar{\delta}+\Phi)}\sum_i m_i\Sigma_{ii},\right] \end{aligned} \tag{27}$$

over the variables Φ and $\Sigma\in \mathrm{SU}(3)$. Here $|\Omega\rangle$ denotes a point in the continuum of the vacua of the massless QCD, and the quark condensates in $|\Omega\rangle$ are given by

$$\langle\Omega|\bar{\psi}_{L,i}\psi_{R,j}|\Omega\rangle = |\Delta|\Sigma_{ij}e^{i\Phi}. \tag{28}$$

Since by definition m_i are positive, it is trivial to see that the potential has a minimum at $\Phi=\Phi_m$, $\Sigma=\Sigma^{(m)}$, where

$$\Phi_m = -\bar{\delta},\ \ \Sigma^{(m)} = I. \tag{29}$$

This is the quark-mass dependence of the quark condensates (16) in the true vacuum. Note that the phase of the quark condensates cancels exactly the CP violating phase $\bar{\delta}$ in the QCD Lagrangian. Since the low energy CP violation occurs only through the combined phase of the quark condensates and the QCD Lagrangian this shows no CP violation at low energies.

For low energy QCD Lagrangian, we notice

$$\langle\Omega_m|\mathcal{L}'_m|\Omega_m\rangle = 2\sum_i m_i|\Delta| \tag{30}$$

from Eqs. (16) and (29), and hence, as far as low energy physics is concerned, the quark mass term $\mathcal{L}'_m$ takes the following form in the true vacuum $|\Omega_m\rangle$:

$$\mathcal{L}''_m = \sum_i m_i \bar{\psi}_i \psi_i \tag{31}$$

where the quark fields ψ_i are normalized as

$$\langle \Omega_m | \bar{\psi}_{L,i} \psi_{R,j} | \Omega_m \rangle = |\Delta| \delta_{ij}. \tag{32}$$

Therefore, the QCD Lagrangian for low energy physics can be written as

$$\mathcal{L}_{\rm QCD} = \mathcal{L}_0 + \mathcal{L}''_m, \tag{33}$$

which evidently shows that the physics is independent of $\bar{\delta}$ and CP is conserved. This resolves the strong CP problem.

Now, what would be the δ dependence of the Schwinger model? For the essentially same reason given for the dynamical alignment of QCD θ vacuum, the θ vacuum alignment by fermion mass term should occur in the Schwinger model too. It is then easy to see the θ vacuum alignment requires $\sigma_0 = -\delta/\sqrt{4\pi}$ in (20), and so δ disappears from the theory. The physics is thus independent of δ.

Finally, we briefly comment on the physical significance of the CP violating phase $\bar{\delta}$ in (5) at high energies. Although this phase become irrelevant at low energies, in chirally symmetric phase where no quark condensation occurs there would be no θ vacuum alignment, and so the CP violation by $\bar{\delta}$ would become observable. This then could be a potential source of CP violation that might become important in physics involving CP violation, for example, such as electroweak baryogenesis.

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